Math 150, *Lecture Notes- Bonds* **Name____________**

Section 1.2 *Finding Limits Graphically and Numerically*

Informal definition of limit: If $f(x)$ become arbitrarily close to a single number L as x approaches c from either side, the **limit** of $f(x)$ as x approaches c is L.

The limit is written as $\lim_{x \to c} f(x) = L.$.

Complete the tables and use the result to estimate the limits. Use a graphing utility to graph the functions and confirm your results.

Ex.1

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{1}{4} = 0.25$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	0.256	0.251	0.250	0.250	0.249	0.244

Rounded to the nearest thousandths

Ex.2

$$\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5} = -\frac{1}{6} = -0.1\bar{6}$$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
$f(x)$	-0.166	-0.167	-0.167	-0.167	-0.167	-0.167

Rounded to the nearest thousandths

Ex.3

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.998	1	1	1	1	0.998

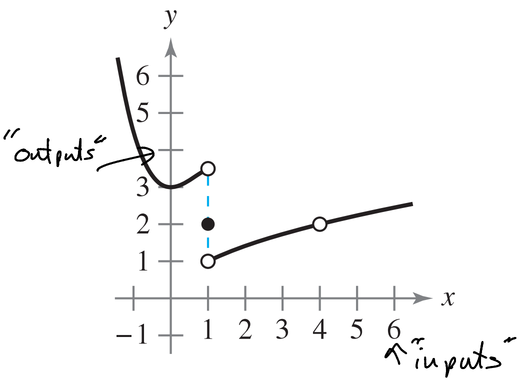
Rounded to the nearest thousandths

Limits That Fail to Exist

Common Types of Behavior Associated with Nonexistence of a Limit

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

Use the graph of f to find the following limits and function values. If the limit does not exist, explain why.



Ex.4 **(a)** $\lim\limits_{x\to 4} f(x)$, **(b)** $\lim\limits_{x\to 1} f(x)$, **(c)** $f(1)$ and **(d)** $f(4)$,

(a) $\lim\limits_{x\to 4} f(x) = 2$

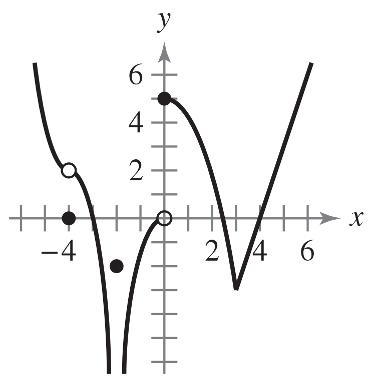
(b) $\lim\limits_{x\to 1} f(x) =$ Does Not Exist

$\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$

(c) $f(1) = 2$

(d) $f(4) =$ Undefined

Use the graph of g to find the following limits and function values. If the limit does not exist, explain why.



limit as an "output expectation" / near a particular input value

Ex.5 **(a)** $\lim_{x\to 3} g(x)$, **(b)** $\lim_{x\to 0} g(x)$, **(c)** $\lim_{x\to -4} g(x)$, **d)** $\lim_{x\to -3} g(x)$,
(e) $g(0)$, **(f)** $g(-3)$, and **(g)** $g(-4)$,

(a) $\lim_{x\to 3} g(x) =$ -3

(b) $\lim_{x\to 0} g(x) =$ Does Not Exist
$\lim_{x\to 0^-} g(x) \neq \lim_{x\to 0^+} g(x)$

(c) $\lim_{x\to -4} g(x) =$ 2

(d) $\lim_{x\to -3} g(x) =$ 0

(e) $g(0) =$ 5

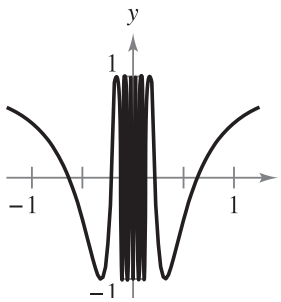
(f) $g(-3) =$ 0

(g) $g(-4) =$ 0

Use the graph to find the following limit. If the limit does not exist, explain why.

$$\lim_{x \to 0} \cos \frac{1}{x}$$

The limit does not exist due to oscillation.



Ex.5 $\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$

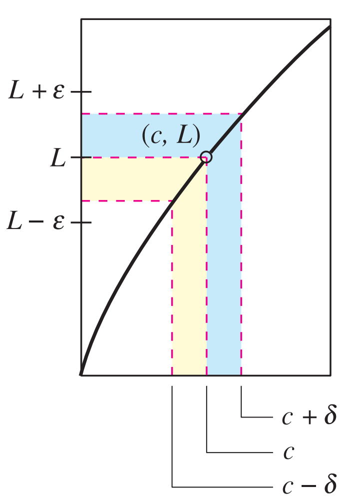
Definition of Limit

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$$0 < |x - c| < \delta, \quad \text{then} \quad |f(x) - L| < \varepsilon.$$



The ε-δ definition of the limit of $f(x)$ as x approaches c